Critical Disorder and Phase Transitions in Random Diode Arrays

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Random diode arrays represent a new class of nonlinear disordered systems related to the physics of thin-film semiconductor structures and some others. When a disorder strength grows through a certain critical value, they undergo a phase transition from almost uniform to strongly nonuniform random electric potential. A piecewise continuous topography of random potential is predicted.

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This Letter introduces a new class of disordered systems, random diode arrays (RDA), that model essential physics of large area noncrystalline semiconductor devices, such as photovoltaics, liquid crystal display drivers, and light emitting panels. The physics of such devices is strongly determined by the material inhomogeneity and the presence of potential barriers in *p*-*n* and other junctions [1]. Correspondingly, a prototype RDA is a set of random photodiodes connected in parallel through a resistive electrode. The universality of the photodiode model is that it combines unidirectional transport with the electric current that can be finite at zero bias [see Eq. (1) below]. Other possible applications of such RDA include charge transport across monomolecular layers where an organic molecule can act as a diode [2], electrically coupled light emitters [3], stochastic heterostructures in nanotubes [4], and spin-polarized electron transport through a domain wall [5].

Each diode in RDA has the ideal current-voltage (j-V) characteristic

$$j = j_0\{\exp[e(V - V_{\rm oc})/kT] - 1\}, \tag{1}$$

where kT and e have their standard meaning, and j_0 and $V_{\rm oc}$ are local diode parameters. The open-circuit voltage ($V_{\rm oc}$) fluctuations have an exponentially strong effect on RDA currents and are considered the main nonuniformity source. A simple nonrestrictive example is a bimodal $V_{\rm oc}$ distribution representing identical weak (low $V_{\rm oc}$) diodes embedded in the uniform matrix of more robust units. Both one-dimensional (1D) and 2D cases are of practical interest. They are topologically the same and can be pictured as a set of random diodes connecting two parallel resistive wires (1D) or parallel resistive sheets (2D).

From the theoretical standpoint, the problem is to describe the topography and statistics of random electric potential in RDA and predict its integral characteristics. This nonlinear problem requires approaches beyond the standard theory of disordered systems. One such approach is developed below for the case of infinitely large RDA with uncorrelated disorder.

Qualitative analysis.—A nontrivial part of the problem is that microdiodes interact with each other by forcing currents through a resistive electrode. On intuitive grounds, the interaction is characterized by the correlation length

$$L_0 = \sqrt{kT/e\rho j_0}. \tag{2}$$

Its physical meaning is that the characteristic thermal fluctuation in the electric potential $\delta V = kT/e$ is balanced by the potential drop $j_0L_0^2\rho$ across the electrode of linear dimension L_0. For $D = 1$, $L_0\rho$ and j_0L_0 represent the resistance and current, and ρ is the resistance per unit length. For $D = 2$, ρ is the sheet resistance and the current is $j_0L_0^2$. For stronger fluctuations $\delta V > kT/e$ the correlation length is even larger, $L = L_0\sqrt{e|\delta V|/kT} > L_0$. L_0 and L vary over a wide range depending on the system parameters and can be macroscopically large (see the numerical estimates below).

We assume in what follows that the characteristic diode size in RDA is small, $l \ll L$ (see the numerical estimates below) and thus a large number $[(L/l)^D \gg 1]$ of random diodes contribute to RDA characteristics. Consider, for simplicity, a bimodal $V_{\rm oc}$ distribution with low weak diode concentration, $(l/R)^D \ll 1$, where R is the average nearest weak diode distance. The average RDA potential is determined by the condition that the sum of all currents given by Eq. (1) vanishes,

$$\overline{V} = -(kT/e)\ln\langle\exp(-eV_{\rm oc}/kT)\rangle, \tag{3}$$

where the angle brackets stand for disorder averaging. Weak diodes find themselves under forward bias $\overline{V} - V_{\rm oc} > 0$ and exponentially large positive currents. Strong diodes ($\overline{V} - V_{\rm oc} < 0$) run negative currents $\approx j_0$. Equation (3) states that weak diodes can have an exponentially strong effect.

The relative dispersion ξ_D in the weak microdiode currents turns out to be a significant parameter characterizing the disorder strength. It can be estimated by noting that each weak diode consumes an exponentially strong relative current $\exp[e(\overline{V} - V_{\rm oc})/kT]$ and that in the region of linear size L_0 the relative fluctuation in their number $\delta N/N \sim 1/\sqrt{N} \sim (R/L_0)^{D/2}$. This gives

$$\xi_D \sim (R/L_0)^D \exp[2e(\overline{V} - V_{\rm oc})/kT]. \tag{4}$$

As verified below, the disorder is strong when $\xi_D \gtrsim 1$.

Parameter estimates.—Practically required transparent electrodes in RDA related devices cannot be made of a "good" metal; hence, substantial sheet resistance $\rho \sim 10\ \Omega/\square$ [6]. The current density is $j_0 \sim (1-3) \times 10^{-2}\ \mathrm{A/cm^2}$ (under one sun illumination) for most photovoltaics [6]. For the room temperature, this yields $L_0 \sim 1$ mm; however, L_0 is by the factor of 100 larger under the ambient room light (lower j_0). Hence, random diodes interact across macroscopic distances. Note that such length scales comparable to the device element sizes may cause mesoscopic effects. For the main fluctuating parameter V_{oc} it is typical to have $\langle eV_{oc}/kT\rangle \sim 10$–30 with the characteristic relative fluctuation of $\sim$0.1–0.3 (see [7] and references therein). The weak diode distance R is not well-known and can be in the range of tens to hundreds of microns [7]. Substituting this into Eq. (4) shows that both the cases of strong ($\xi_D \gg 1$) and weak ($\xi_D \ll 1$) disorder are realistic.

Quantitative approach.—The electric potential distribution in RDA can be described based on the diode equation (1) and Ohm's law:

$$\nabla \mathbf{i} = -j, \qquad \rho \mathbf{i} = -\nabla V, \tag{5}$$

where $\mathbf{i}$ is the lateral current (current density) in the resistive electrode for $D = 1$ ($D = 2$), V is the electric potential, and j_0 is the specific transversal currents (per length for $D = 1$ or per area for $D = 2$) defined in Eq. (1). Introducing the dimensionless units

$$\phi = e(V - \overline{V})/kT, \qquad y = x/L_0, \tag{6}$$

Eqs. (5) can be reduced to the form

$$\nabla^2 \phi = (1 + \zeta)\exp(\phi) - 1, \tag{7}$$

where ζ is a random variable,

$$\zeta = \exp[e(\overline{V} - V_{oc})/kT] - 1, \qquad \langle \zeta \rangle = 0. \tag{8}$$

Its statistics is described by the correlation function

$$\langle \zeta(0)\zeta(\mathbf{r})\rangle = B\delta(\mathbf{r}), \qquad B = \text{const.} \tag{9}$$

Here $\delta(\mathbf{r})$ is the delta function of the coordinate $\mathbf{r}$. [Because of the microdiode finite size, $\delta(\mathbf{r})$ should be understood as having a small yet finite width l].

In what follows ϕ is presented as a superposition of the short-range and long-range components,

$$\phi = \phi_s + \phi_L, \qquad |\phi_s| \ll 1, \qquad \langle \phi_s \rangle = 0. \tag{10}$$

ϕ_s has the characteristic space scale $l \ll 1$. Its amplitude is assumed to be small, $\varphi_s \ll 1$, since the neighboring microdiodes are separated by small electrical resistance; the corresponding condition is derived in Eq. (19) below. The long-range component is not necessarily small and is approximately constant on the scale of l.

Linearizing Eq. (7) in $|\phi_s| \ll 1$ and averaging over a region of linear dimension x, such that $l \ll x \ll 1$, yields

$$\nabla^2 \phi_L = (1 + \langle \phi_s \zeta \rangle_x)\exp(\phi_L) - 1. \tag{11}$$

In accordance with the central limit theorem, a random quantity $\langle \phi_s \zeta\rangle_x$ can be represented as $f(x)\langle \phi_s \zeta\rangle$, where the latter average is taken over an infinitely large volume ($x \to \infty$) and $f(x)$ obeys the Gaussian statistics with the average $\langle f \rangle = 1$. Its fluctuations are small, $\delta f \sim (l/x)^D$, since the averaging is taken over a large number of microdiodes, $(x/l)^D \gg 1$.

Eliminating the terms absorbed by Eq. (11) and neglecting ϕ_s in its right-hand side (rhs), linearized Eq. (7) becomes

$$\nabla^2 \phi_s = \zeta \exp(\phi_L), \tag{12}$$

where ϕ_L is considered constant. A system of coupled equations (11) and (12) describe the long-range and short-range components of the electric potential.

The quantity $\langle \phi_s \zeta\rangle_x$ in Eq. (11) can be expressed through the correlation function $\langle \zeta(0)\phi_s(r)\rangle$ with $r = l$. Multiplying Eq. (12) by $\zeta(0)$ and then averaging gives the equation

$$\nabla^2 \langle \zeta(0)\phi_s\rangle_x = Bf(x)\delta(\mathbf{r})\exp(\phi_L), \tag{13}$$

whose solution is

$$\langle \zeta(0)\phi_s\rangle_x = \frac{Bf\exp(\phi_L)}{2\pi}\begin{cases} \pi|r| + C_1 & \text{for } D = 1, \\ \ln r + C_2 & \text{for } D = 1, \end{cases} \tag{14}$$

where constants C_1 and C_2 must be determined from the boundary conditions. Because Eq. (12) is restricted to the region $r \ll 1$, the standard boundary condition is hard to impose. Offering an alternative is the observation that, in the absence of other characteristic lengths, the correlation between ζ and ϕ_s should decay over distances $r \sim 1$. We approximate it by $\langle \zeta(0)\phi_s(r = 1)\rangle_x = 0$, which yields $C_1 = -\pi$ and $C_2 = 0$. [The latter analysis of $\langle \zeta(0)\phi_s\rangle$ is readily verified for a small disorder when Eq. (7) is linear in ϕ.] Substituting into Eq. (14) $r = l$ yields

$$\nabla^2 \phi_L = -\frac{1}{4\xi}[\exp(\phi_L + \ln 2\xi) - 1]^2 + \frac{1}{4\xi} - 1, \tag{15}$$

where

$$\xi = \frac{Bf}{2}\begin{cases} 1 & \text{for } D = 1, \\ (1/\pi)\ln(1/l) & \text{for } D = 2. \end{cases} \tag{16}$$

One immediate result of the above analysis is that there exists a critical disorder strength $\xi_c = 1/4$, such that the electric potential and current distributions are qualitatively different for the cases of $\xi < \xi_c$ and $\xi > \xi_c$. In the case of $\xi < \xi_c$ one can calculate the average potential in the system by setting the left-hand side zero in Eq. (15),

$$\langle \phi_L \rangle = \ln\left(\frac{1 - \sqrt{1 - 4\xi}}{2\xi}\right). \tag{17}$$

This solution fails when $\xi > 1/4$. Furthermore, analyzing the corrections $\delta\phi_L \equiv \phi - \langle\phi\rangle$ by the perturbation technique, it is straightforward to see from Eq. (15) that the characteristic length scale and amplitude of nonuniformities diverge as $\xi \to \xi_c$. Hence, ξ is a figure of merit for the nonuniformity effects and characterizes the disorder strength. Estimating B as defined in Eq. (9) for a bimodal V_{oc} distribution and substituting it into Eq. (16) leads to $\xi \sim \xi_D$ with ξ_D [from Eq. (4)] representing the relative dispersion microdiode currents. Below we consider the cases of subcritical ($\xi < \xi_c$) and supercritical ($\xi > \xi_c$) disorder separately.

Subcritical disorder.—Small fluctuations of random function $f(x)$ and corresponding variations $\delta\xi$ become an important source of randomness for small $\xi \ll 1$ when the rhs in Eq. (15) is dominated by the contribution that is inversely proportional to ξ. Because $\delta\xi$s are small, so are the variations in ϕ_L (the system is almost uniform). They satisfy the linearized equation (15),

$$\nabla^2\delta\phi_L = \langle\phi\rangle\sqrt{1-4\xi}\delta\phi_L - \delta\xi\exp\langle 2\phi\rangle \qquad (18)$$

and thus obey the Gaussian statistics.

Supercritical disorder.—For $\xi > \xi_c$, the rhs of Eq. (15) is everywhere negative. The negative curvature $\nabla^2\phi$ exponentially increases in the absolute value with ϕ above its maximum $\phi_m = \ln(1/2\xi)$. Therefore, the spectrum of ϕ cannot span much beyond ϕ_m. Indeed, any increase in ϕ (i.e., positive $\nabla\phi$) would be strongly limited by exponentially large negative $\nabla^2\phi$. For $\xi \gg \xi_c$ and $\phi < \phi_m$ we find $\nabla^2\phi \approx -1$; that is, $\phi(\mathbf{r})$ is close to a negative curvature paraboloid and is unbounded below. This is consistent with the above observation that the average $\langle\phi\rangle$ is not defined when $\xi > \xi_c$.

The unbounded spectrum appears in the framework of the approximation employed. The lower boundary effects can be included beyond that approximation by accounting for the lowest ϕs that correspond to the weakest diodes. In the above approximation framework, the weakest diode appears as a singularity where $\nabla\phi$ undergoes a finite change and the electric potential cannot be decomposed into a sum of long- and short-range components. Taking such singularities into account, the potential has a piecewise continuous structure formed by a set of negative curvature paraboloids (far from weak diodes where the approximation of smoothly varying potential is valid), connected in a singular way at weak diodes (see Figs. 2 and 3).

The location of singularities needs to be further specified if the V_{oc} distribution is not a bimodal. A diode weakest in its screening length neighborhood ($V_{oc} = V_{oc,min}$) will obviously cause a singularity. On physical grounds, a less weak diode at distance r in the same neighborhood will cause a singularity if it is a local current sink. This happens when the difference between its V_{oc} and $V_{oc,min}$ is smaller than the electric potential drop $j_0\rho r^2$ across the resistive electrode between the two diodes. This understanding is consistent with the results of numerical modeling (Figs. 2 and 3).

Applicability.—The linearization of Eq. (7) with respect to ϕ_s remains valid when $\langle\phi_s^2\rangle \ll 1$. Multiplying Eq. (12) by $\phi_s(0)$, averaging, and taking into account Eqs. (14) and (16), reduces the latter condition to

$$\langle\phi_s^2\rangle = \xi\exp(2\phi_L) \ll 1. \qquad (19)$$

It is obviously satisfied for the case of subcritical disorder, $\xi \ll 1$. For $\xi \gg 1$, we take into account that the spectrum of ϕ_L is confined to the region $\phi_L \lesssim \ln(1/2\xi)$. As substituted in Eq. (19), this gives $\langle\phi_s^2\rangle \lesssim 1/4\xi \ll 1$; hence, the inequality in Eq. (19) obeys in the far supercritical region. However, it fails in the critical region.

Numerical simulations.—As a verification, 1D and 2D RDAs of 1000 diodes have been simulated numerically. The individual diode V_{oc}s were randomly generated to obey either Gaussian or uniform distributions with desired averages and dispersions. The electric potential and current distributions were then found by numerically solving a set of Kirchhoff's equations with the open-circuit boundary conditions.

For RDA with subcritical disorder the distribution in Fig. 1 has a smoothly varying shape similar to what is typically considered random potential in the existing theory of disordered systems. Such shape, smallness of the potential fluctuations, and their verified Gaussian statistics are consistent with the above analytical predictions.

The results of numerical simulations for 1D and 2D RDA with supercritical disorder are shown in Figs. 2 and 3. They confirm, indeed, the conclusion of piecewise continuous potential distribution of randomly located negative curvature paraboloids forming cusps in connection points.

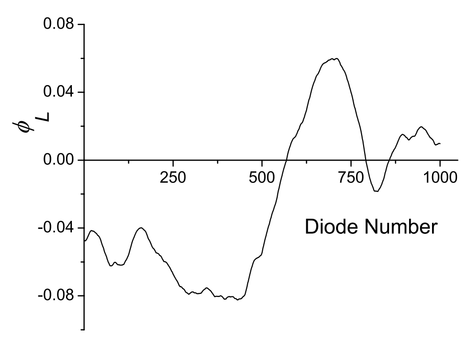

FIG. 1. 1D ϕ_L distribution for the case of subcritical disorder numerically simulated for a random diode circuit with uniformly distributed V_{oc} where $\langle eV_{oc}/kT\rangle = 10$, the relative standard deviation of 10%, and the electrode resistance between the nearest neighbor diodes $r = 10^{-4}kT/qj_0$. The diode number plays the role of the linear coordinate. Note the small amplitude of the fluctuations.

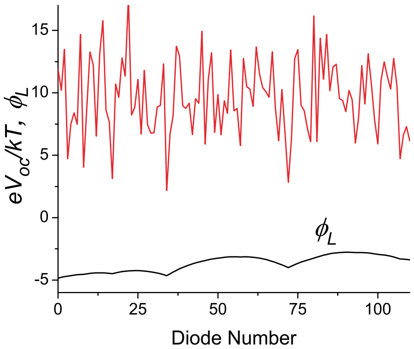

FIG. 2 (color online). V_{oc} and reduced electric potential ϕ distributions for the case of supercritical disorder numerically simulated for a 1D random diode circuit with the Gaussian V_{oc} distribution, $\langle eV_{oc}/kT\rangle = 10$, the relative standard deviation of 30%, and the electrode resistance between the nearest neighbor diodes $r = 0.01kT/qj_0$. Note singular ϕ shapes in the proximity of minima.

Example implication.—As an implication of the above theory consider the tail of the probabilistic distribution of weak diode currents (i.e., $j > 0$) in RDA. A weak diode current is stronger when it does not have equal competitors in a larger domain and thus robs currents generated by a larger number of neighboring robust diodes. The probability of finding no weak diodes in the region of large radius $r > R$ is given by the Poisson distribution $\exp[-(r/R)^D]$. Because the amplitude of electric potential $\delta\phi$ is parabolic in r, we get $\delta\phi \propto r^2$. The electric current can be expressed as $J \sim \delta\phi/[\rho r^{(2-D)}]$, where $D = 1, 2$ [see the discussion after Eq. (2)]. As a result the probability distribution for the current takes the form

$$g(J) \propto \exp(-J/J_0) \quad \text{for } J > J_0, \tag{20}$$

for both the cases of $D = 1$ and $D = 2$, where $J_0 = j_0R^D = \text{const}$ (in conventional units). This prediction agrees well with the results of numerical simulations.

To conclude, RDA represent a new class of nonlinear disordered systems modeling large area semiconductor devices. They are predicted to undergo a phase transition from the state of almost uniform to that of strongly nonuniform electric potential; the corresponding order parameter (figure of merit, ξ) and its critical value are derived. From the practical standpoint, this understanding can serve as a guide for technology to mitigate the nonuniformity effects. The established piecewise continuous topography of random potential represents a new concept in the physics of nonlinear disordered systems. The above consideration leaves many important questions unanswered. Those of the role of the boundary conditions for finite RDA, their j-V characteristics, mesoscopic effects, and properties of other topologies seem to be the most challenging.

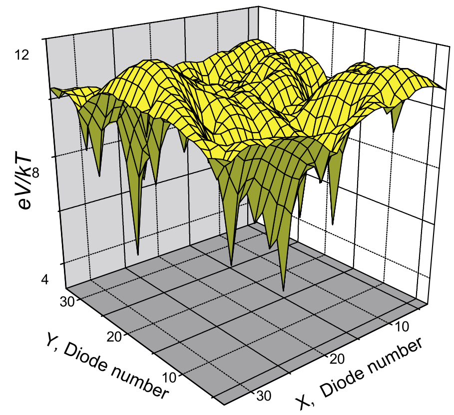

FIG. 3 (color online). Electric potential distribution for the case of 2D supercritical disorder numerically simulated for RDA of 31×31 diodes with the Gaussian V_{oc} distribution, $\langle eV_{oc}/kT\rangle = 16$, the relative standard deviation of 30%, and the electrode resistance between the nearest neighbor diodes $r = 0.01kT/qj_0$. Note cusps in the proximity of minima and paraboloidal shapes far from them.

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